**Dynamic Manipulation of *n*-Revolute-Joint Planar Manipulators**

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**1. Problem Statement**

In class project 5, our group pursues the task to develop a planning tool for an *n*-revolute joint planar manipulation systems in OMPL. The planning tool takes a variety of input parameters (joint number, *n*, link lengths and masses) specified by the users to compute the joint torques required to achieve a pose and to generate solutions to motion planning problems.

Our group consists of an electrical engineer and two mechanical engineers, all of whom have strong academic interest in robotic manipulation systems. Precision manipulation is of paramount importance for all robots and has always been a difficult challenge. It adds more complication to the challenge when there is the need to plan motions for the robots. Not only do the motion plans have to satisfy the user queries, but they also have­­ to respect the differential (kinematic and dynamical) constraints. Planning the path while respecting the differential constraints is a two-point boundary value problem (BVP) which is computationally expensive to solve. Our group is intrigued by the challenge and is determined to find a solution to the problem. Over the course of the class, we have gained exposure to motion planners using random sampling. These planners can by-pass the costly 2-point BVP and are highly effective in handling planning with differential constraints. Our group is highly motivated by the prospect of incorporating precision dynamic manipulation into motion planning.

**2. Methods**

The dynamic manipulation of an *n*-revolute-joint planar robot involves solving ordinary differential equations (ODE) in the general form of:

where in the context of this problem,

, , are the joint angle, angular velocity and angular acceleration of the ith joint respectively

, the input control to the ODE, in the context of this problem is the applied torque to each link

More detailed ODEs are set up using the Lagrangian formulation of the dynamics model of the rigid manipulator.

where

is the inertia matrix

is the vector of Coriolis, centrifugal forces

is the viscous friction matrix. In the scope of this project, it is assumed the joints are frictionless, hence = 0

is the vector of gravity force

The governing ODE can now be written as:

The above ODE have render the dynamic manipulation problem solvable by the ompl::control::ODESolver function. The state space of the *n*-revolute-joint planar robot is set up as the Cartesian product of *n* SO2 state space. In OMPL, it is expressed as SO2 + SO2 + … + SO2. The control space of the robot is set up as the Cartesian product of *n* R1 state space.

The RRT planner is chosen to perform the motion planning tasks. RRT planner is an effective tool to handle motion planning with differential constraints, as it utilizes random sampling to generate probable control set and use the probable control set to propagate tree states rapidly towards goal because of the goal bias. The effectiveness in dealing with differential constraints is due to the fact that the random sampling help by-pass the computationally costly solving of 2-point BVP. The planner is considered *Anytime* for its relative quickness to find the solution.

The comprehension of the underlying mathematics of the dynamics manipulation problem and the knowledge of these key attributes of RRT planner afford us the confidence about our approach to solve the problem.

A visualization tool is designed in MATLAB to provide graphical representation of the motion planning solution. It requires only the input of joint angles and link lengths by the users. The figure below is an example execution (unrelated to the experiments conducted in this study) of the visualization tool to illustrate the effectiveness of the visualization tool.



Figure 1: Visualization tool created in MATLAB. 10 random poses (left) and 1000 random poses for a 3-R planar robot with all link length equal to 0.5

**3. Experiments and Results**

**4. Analysis**